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## Testing cosmological supersymmetry breaking

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Banks has proposed a relation between the scale of supersymmetry breaking and the cosmological constant in de Sitter space. His proposal has a natural extension to a general FRW cosmology, in which the supersymmetry breaking scale is related to the Hubble parameter. We study one consequence of such a relation, namely that coupling constants change as the universe evolves. We find that the most straightforward extension of Banks' proposal is disfavored by experimental bounds on variation of the fine structure constant.

# 1 Introduction

The Holographic Principle [1] states that the total number of degrees of freedom in a theory of quantum gravity scales like the surface area. This is radically different from the behavior of quantum field theory, in which the number of states at high energy scales like the volume. This suggests that the usual field theory calculation of divergent radiative corrections to scalar masses is modified by quantum gravity.

Moreover, holographic theories have a UV/IR connection [2], which relates high energies to long distances. In particular the spectrum of high energy states in a holographic theory may well be determined by the large scale structure of the universe. Putting these ideas together, it seems plausible that in a holographic theory, scalar masses are related to cosmology.

Banks has put forward a very concrete proposal for such a relation [3]. He considers M-theory in a de Sitter background, with a cosmological constant  $\Lambda > 0$  corresponding to a vacuum energy density

$$\rho_{\text{vac}} = m_{\text{vac}}^4 = \frac{\Lambda}{8\pi G}.$$

The de Sitter geometry breaks supersymmetry. Banks proposes that the supersymmetry breaking scale is not given by the naive guess  $m_{\text{susy}} \approx m_{\text{vac}}$ . Rather, he suggests that UV/IR effects could enhance this to

$$m_{\text{susy}} \approx (m_{\text{planck}} m_{\text{vac}})^{1/2}.$$

With a Planck mass of  $10^{19} \text{ GeV}$  and a vacuum energy of  $10^{-3} \text{ eV}$ , this leads to a phenomenologically acceptable breaking of SUSY at the few  $\text{TeV}$  scale.

In this paper we generalize Banks' proposal to a general flat FRW cosmology (section 2). The natural generalization relates the SUSY breaking scale to the Hubble parameter. Via the renormalization group a change in the SUSY breaking scale affects low energy coupling constants, so coupling constants will change as the universe evolves. We consider the experimental bounds on variation of the fine structure constant in section 3, and show that the simplest extension of Banks' proposal is experimentally disfavored. Section 4 contains our conclusions.

## 2 Cosmological supersymmetry breaking in an FRW universe

Banks has proposed that in a de Sitter background the scale of supersymmetry breaking is set by

$$m_{\text{susy}} \approx m_{\text{planck}}^{1/2} \left( \frac{\Lambda}{8\pi G} \right)^{1/8}.$$

This formula is supposed to be a consequence of the finite number of states which are available in a de Sitter background. This motivates us to begin our search for an appropriate generalization of Banks' formula by rewriting  $m_{\text{susy}}$  in terms of the entropy of de Sitter space  $S = 3\pi/G\Lambda$ .

$$m_{\text{susy}} \approx m_{\text{planck}}^{1/2} \left( \frac{3}{8G^2 S} \right)^{1/8} \quad (1)$$

This relation has a natural extension to a general spacetime: *the supersymmetry breaking scale is given by (1), with  $S$  interpreted as the holographic bound on the cosmological entropy.*

For the remainder of this paper, we specialize to flat ( $k = 0$ ) FRW universes, with metric

$$ds^2 = -dt^2 + R^2(t) (dr^2 + r^2 d\Omega_2^2)$$

and Hubble parameter  $H = \dot{R}/R$ . Following [4, 5] we take the cosmological entropy to be bounded by the area of the apparent horizon. The apparent horizon is a sphere of radius  $r_{AH} = 1/HR$  and area  $A_{AH} = 4\pi/H^2$ , corresponding to an entropy

$$S = \frac{A_{AH}}{4G} = \frac{\pi}{GH^2}. \quad (2)$$

Thus the SUSY breaking scale is related to the Hubble parameter by

$$m_{\text{susy}} \approx m_{\text{planck}}^{1/2} \left( \frac{3H^2}{8\pi G} \right)^{1/8}. \quad (3)$$

Curiously, the quantity in parenthesis is the critical density. For our purposes, it is more convenient to rewrite this as a ratio, with a subscript 0 denoting the present.

$$\frac{m_{\text{susy}}(t)}{m_{\text{susy}}(t_0)} = \left( \frac{H(t)}{H_0} \right)^{1/4} \quad (4)$$

This implies that the supersymmetry breaking scale changes with time as the universe evolves.

Although the scale of supersymmetry breaking has not been directly observed, we can put limits on any possible changes in  $m_{\text{susy}}$  because the scale of supersymmetry breaking affects the values of the low-energy coupling constants. This dependence on  $m_{\text{susy}}$  arises from the renormalization group, which states that at one loop gauge couplings  $\alpha \equiv g^2/4\pi$  evolve with scale according to

$$\frac{1}{\alpha(\mu^2)} - \frac{1}{\alpha(M^2)} = \frac{b_0}{2\pi} \log \frac{\mu}{M}.$$

The  $\beta$ -functions generally change at the scale where supersymmetry is broken, and this makes the low-energy couplings sensitive to the value of  $m_{\text{susy}}$ .

The precise dependence on  $m_{\text{susy}}$  is easily obtained. The Hubble parameter has been decreasing as the universe evolves, so the supersymmetry breaking scale (3) has been decreasing with time. Suppose that in the course of this evolution the supersymmetry breaking scale drops from an initial value  $m_1$  at time  $t_1$ , to a new value  $m_2$  at time  $t_2$ . Above the scale  $m_1$  we assume that couplings are not affected by cosmology, so the couplings are identical at the two different times:

$$\alpha(m^2)|_{t_1} = \alpha(m^2)|_{t_2} \quad \text{for } m > m_1.$$

We also assume that below the scale  $m_2$  the runnings are the same:

$$b_0|_{t_1} = b_0|_{t_2} \quad \text{below the scale } m_2.$$

However in the intermediate range  $m_2 < m < m_1$  the  $\beta$ -functions are different at the two different times. This leads to a change in the value of the observed couplings at low energy, which is given by the difference in the two  $\beta$ -functions.

$$\frac{1}{\alpha}|_{t_2} - \frac{1}{\alpha}|_{t_1} = \frac{1}{2\pi} (b_0^{SM} - b_0^{MSSM}) \log \frac{m_1}{m_2} \quad (5)$$

We now specialize to the evolution of the fine structure constant. In the standard model the photon is a mixture of the  $U(1)_Y$  and  $SU(2)_L$  gauge fields, with coupling

$$\frac{1}{\alpha} = \frac{1}{\alpha_Y} + \frac{1}{\alpha_{SU(2)}}.$$

The appropriate one-loop beta functions in the standard model and its supersymmetric extension are therefore given by [6]

$$b_0 = b_0^Y + b_0^{SU(2)} = \begin{cases} -10/3 & \text{standard model} \\ -12 & \text{MSSM} \end{cases} \quad (6)$$

This implies that the fine structure constant depends on time according to

$$\frac{1}{\alpha(t_0)} - \frac{1}{\alpha(t)} = \frac{13}{3\pi} \log \frac{m_{\text{susy}}(t)}{m_{\text{susy}}(t_0)}.$$

Given the proposed relationship (4) between  $m_{\text{susy}}$  and the Hubble parameter, this implies that

$$\frac{1}{\alpha(t_0)} - \frac{1}{\alpha(t)} = \frac{13}{12\pi} \log \frac{H}{H_0}. \quad (7)$$

Since  $H$  was larger in the past, our proposal implies that  $\alpha$  was larger in the past.

The evolution of the Hubble parameter is determined by the Friedmann equation

$$H^2 = \frac{8\pi G}{3} \sum_i \rho_i$$

where  $\rho_i$  are the various components of the energy density. We will model the universe as dominated by matter plus vacuum energy. This leads to an equation for the evolution of the normalized scale factor  $a(t) = R(t)/R_0$ ,

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 (\Omega_\Lambda + \Omega_M a^{-3}) \quad (8)$$

where  $\Omega_\Lambda$ ,  $\Omega_M$  are the present-day fractions of the critical density. Thus the Hubble parameter is given by

$$\frac{H}{H_0} = \sqrt{\Omega_\Lambda + \frac{\Omega_M}{a^3}}. \quad (9)$$

We also have a relation between the scale factor and the age of the universe,

$$t = \frac{1}{H_0} f(a) \quad (10)$$

where

$$f(a) = \frac{2}{3\sqrt{\Omega_\Lambda}} \log \left[ \left( \frac{\Omega_\Lambda a^3}{\Omega_M} \right)^{1/2} + \sqrt{1 + \frac{\Omega_\Lambda a^3}{\Omega_M}} \right].$$

### 3 Experimental bounds

Experimental constraints on the time variation of  $\alpha$  come from a variety of sources. Direct lab measurements were performed by [7] using clocks based on ultra-stable atomic oscillators. By comparing rates from different clocks, the authors obtained a bound

$$|\dot{\alpha}/\alpha| \leq 3.7 \times 10^{-14} \text{ yr}^{-1}. \quad (11)$$

Equations (7) and (9) predict that at the present time

$$\begin{aligned}\dot{\alpha}/\alpha &= -\frac{13}{8\pi}\alpha\Omega_M H_0 \\ &= (-8.7 \pm 1.8) \times 10^{-14} \text{ yr}^{-1}\end{aligned}\tag{12}$$

where we have used the values  $\Omega_M = 0.3$ ,  $H_0 = h/(9.78 \times 10^9 \text{ yr})$  and the uncertainty corresponds to varying  $h$  from 0.6 to 0.9 [8]. This is about a factor of two larger than the experimental bound (11).

A much more stringent bound comes from the Oklo reactor, a natural nuclear reactor which was triggered about 1.8 billion years ago. From an analysis of this phenomenon, the authors of [9] obtained a bound

$$-0.9 \times 10^{-7} < \frac{\Delta\alpha}{\alpha} < 1.2 \times 10^{-7}.\tag{13}$$

Equations (7), (9), (10) predict an effect which is three orders of magnitude larger:

$$\frac{\Delta\alpha}{\alpha} \equiv \frac{\alpha(t) - \alpha_0}{\alpha_0} = (1.9 \pm 0.4) \times 10^{-4}.$$

Again the quoted uncertainty corresponds to  $\Omega_M = 0.3$  and  $h$  ranging from 0.6 to 0.9. This seems like a disaster for our proposal (4), but as we discuss in the conclusions, this result should be qualified.

Another bound comes from observation of quasar absorption lines. One group has obtained the bound [10]

$$\frac{\Delta\alpha}{\alpha} = (-4.6 \pm 4.3 \pm 1.4) \times 10^{-5}\tag{14}$$

at redshifts  $z \sim 2 - 3$ , while other groups have reported stronger results [11]. Equations (7) and (9) give

$$\frac{\Delta\alpha}{\alpha} = 2.1 \times 10^{-3}$$

at  $z = 1.5$  (recall  $a = 1/(1+z)$ ). Even compared to the conservative bound (14), this is about two orders of magnitude too large.

Finally, we consider the bound from big bang nucleosynthesis. A recent bound was obtained by [12], who found a limit

$$\frac{\Delta\alpha}{\alpha} = (-7 \pm 9) \times 10^{-3}.$$

At the time of nucleosynthesis the universe was radiation-dominated, with  $H(t) = 1/2t$ . Equation (7) predicts that at the end of nucleosynthesis

$$\frac{\Delta\alpha}{\alpha} = (9.7 \pm 0.1) \times 10^{-2}\tag{15}$$

where we have set  $t_{BBN} = 100 \text{ sec}$ . This is about an order of magnitude larger than the experimental bound.

One might wonder whether we are allowed to apply our formulas to the early universe, since the relation (1) is only expected to be valid for very large values of the de Sitter entropy [3]. In a radiation dominated universe the entropy follows from (2),  $S = 4\pi t^2/t_{\text{planck}}^2$ . Thus  $S \approx 10^{90}$  at the end of nucleosynthesis; presumably this is large enough for our generalization of (1) to hold.

## 4 Conclusions

In this paper we have presented an extension of Banks' cosmological supersymmetry breaking proposal to a flat FRW universe. As we have seen, bounds on variation of the fine structure constant provide a stringent test of this extended proposal: it seems to be ruled out, especially by the Oklo reactor data.

We reached this conclusion by studying the behavior of the fine structure constant, while neglecting the cosmological evolution of all other parameters in the standard model. In a sense this makes our analysis very conservative, since one generally expects that relevant couplings in the standard model should have a power-law dependence on  $m_{SUSY}$ . Given the proposal (4), this would give the relevant couplings a power-law dependence on the Hubble parameter, in gross contradiction with experiment. However, the behavior of relevant couplings may depend on the details of the way in which supersymmetry breaking is communicated to the standard model.

We seem to have found that the proposal (4) can be ruled out just by considering marginal operators. However, this conclusion should be qualified, because there are some potentially important effects at the level of marginal operators that we have ignored. For example, we have neglected the fact that according to our proposal cosmological evolution should also affect the QCD scale. On the one hand, changes in the QCD scale should be tightly constrained by big bang nucleosynthesis. But on the other hand changing the QCD scale may well modify the analysis [9] of the Oklo reactor data, which implicitly assumed that the QCD scale was constant.

To address this concern, let us note that ref. [13] carried out a global fit to numerous observations (including a much more conservative analysis of the Oklo data than [9]). They allowed  $\Lambda_{QCD}$ ,  $G_F$ ,  $\alpha$ ,  $G_N$  and  $m_e$  to vary

independently, and found an upper bound

$$|\dot{\alpha}/\alpha| < 1.4 \times 10^{-15} \text{ yr}^{-1} \quad (95\% \text{ confidence level}).$$

This limit is almost two orders of magnitude smaller than our present-day prediction (12).

It is possible that the supersymmetry breaking scale is determined by cosmology, but not in the way that we have suggested. We took the entropy that appears in (1) to be given by the area of the apparent horizon. This seems quite natural, following [4], but other choices could be contemplated, such as the area of the event horizon. It could also be that the entropy is determined by the value of  $\Lambda$ , even if  $\Lambda$  never plays an important role in the evolution of the universe. Ref. [14] provides some support for this possibility.

Of course, it could be that supersymmetry breaking and cosmology are unrelated, or that such a relation exists but is far more subtle than anything we have discussed here.

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